St. Mary's CVA MATHEMATICS CALCULATION POLICY

November 2023

To be reviewed annually

This policy has been developed by the Maths Subject Leader to facilitate **smoother transition**, **consistency** and **progression** in order to **raise standards** for all pupils.

The development of this policy was informed by the mastery CPD provided by The National Centre for Excellence in Teaching of Maths (NCETM) Maths Hub – East Midlands.

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Aims

The Calculations Policy aims to ensure all pupils:

- ✓ understand important concepts and make connections within mathematics
- ✓ show high levels of fluency in performing written and mental calculations
- ✓ are taught consistent calculation strategies
- ✓ are ready for the next stage of learning
- ✓ have a smooth transition between phases
- ✓ can add, subtract, multiply and divide efficiently
- ✓ are competent in fluency, reasoning and problem-solving.

Rationale

This policy was borne out of analysis of our SATs data showing that for some of the mathematical strands, the strategies that the children were using were not as successful as they could be. It also reflects a change in teaching approach, with the school adopting a mastery approach to mathematics teaching.

To ensure consistency and progression in the teaching of mathematics throughout the school, this Calculations Policy has been produced and developed by the Maths Subject Leader. The strategies in the policy were informed by a range of research and consideration of successful strategies currently in use. This policy explains the different strategies used for calculations in our school from EYFS to KS2.

Children are introduced to the processes of calculations through practical, oral and mental activities. As children begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods. They learn to interpret and use the signs and symbols involved. Children learn how to use models, images and resources, such as empty number lines, to support their mental and informal written methods of calculation.

However, mental calculation is not at the exclusion of written recording and is seen as complementary to, and not separate from it. Each of the four operations: addition, subtraction, multiplication and division, build on mental skills which provide the foundations for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly.

Overview of our approach to calculation strategies

Our approach

We feel that it is fundamental for children to be able to move from conceptual learning to abstract learning in order to be able to successfully understand, use and apply their mathematical skills. The calculation strategies which will be used will reflect this ideology – moving from Concrete to Pictorial and then Abstract recording (CPA), leading to more formal written methods. Mental methods and strategies will work in partnership with these methods.

Selecting the methods

We have considered the following factors when selecting the calculation strategies to be used:

- research and evidence
- building on experiences in EYFS
- consistency and progression across the school
- ability to apply mental methods
- an emphasis on understanding the concept rather than relying on the procedure
- written methods that can be applied across all four operations
- adopting the Concrete, Pictorial and Abstract approach (CPA)
- methods that can be followed through to algebraic representation.

Other methods

We recognise that there are many successful written methods in use today. However, we know from the evidence collected and the research we have been involved in, that the emphasis should remain on understanding the concept (relational learning) and operation rather than procedural (instrumental) learning (Skemp 2012).

Representations

How we represent an idea in maths is a key part of the process by which we develop understanding and give meaning to that idea.
Barmby- Primary Maths: Teaching for Understanding

External representations permit us to talk about mathematical relations and meaning.
Kaput-Theories of Mathematical learning

All schools use a variety of Concrete, Pictorial and Abstract (CPA)representations for numbers and calculations. Pupils should have an opportunity to manipulate and experience a variety of models, images, and resources to enable them to choose the most suitable representation for each calculation. In the picture above you can see some examples of models, images and resources: arrow cards, bead strings, counters, dice, dienes, digit cards, multilink cubes, number fans, number lines, number tracks, Numicon, place value cards, 100 square, sorting objects etc.

Addition +

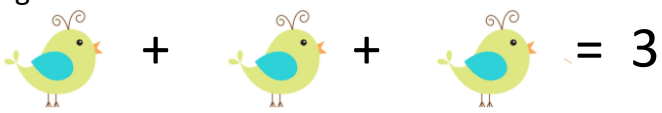
Definition

Addition is the process of calculating the total of two or more numbers or amounts. It is the inverse of subtraction.

Early learning

Using a range of practical resources and real-life contexts, pupils develop their understanding of the concept of addition through counting activities. They then use pictures/diagrams to represent the calculation.

E.g. There are 2 birds. Another bird flies in. How many are there **altogether**?



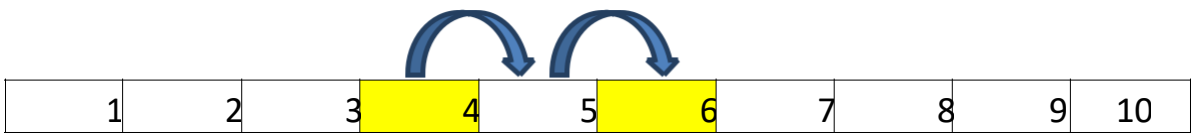
Mental Calculations

- Counting forwards and backwards
- Understanding that addition is **commutative**.
- **Partitioning**
- Recalling number bonds
- Using subtraction as the **inverse** of addition

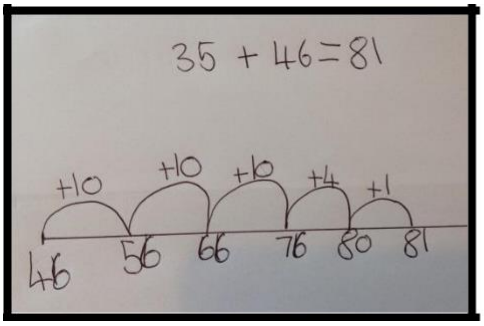
Written Methods

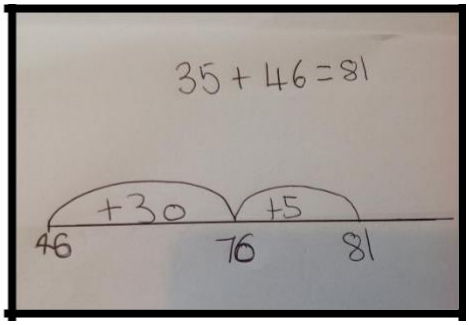
Number tracks/lines

- Using a number line to add e.g. 4 + 2
- Begin with a number track:

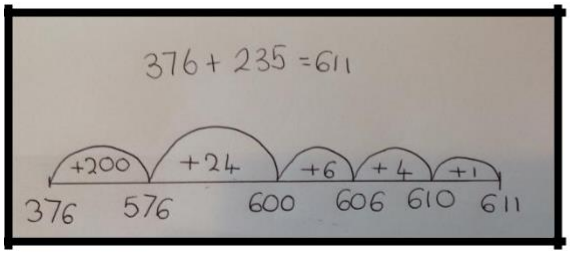


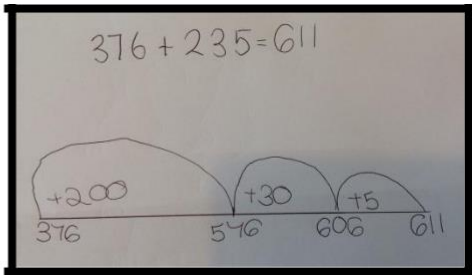
Progress to an empty number line using more efficient strategies. E.g. 35 + 46



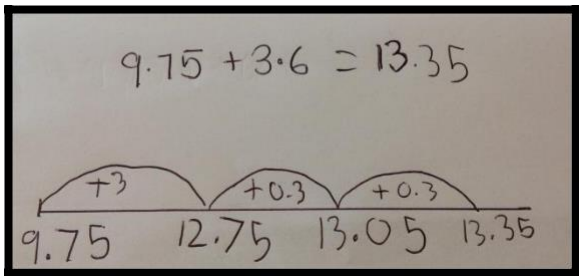


E.g. 376 + 235



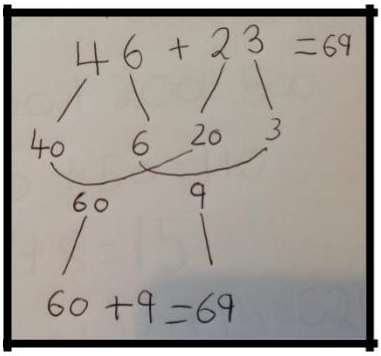


E.g. 9.75 + 3.6

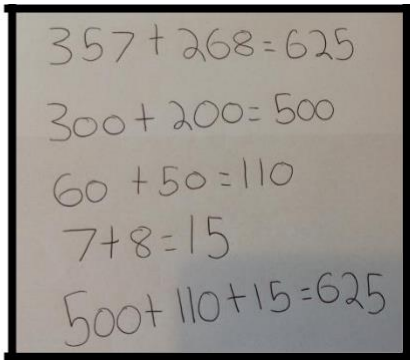


Partitioning two digit numbers and then three digit numbers.

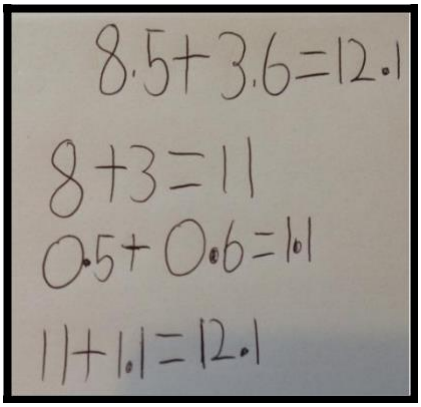
E.g. 46 + 23



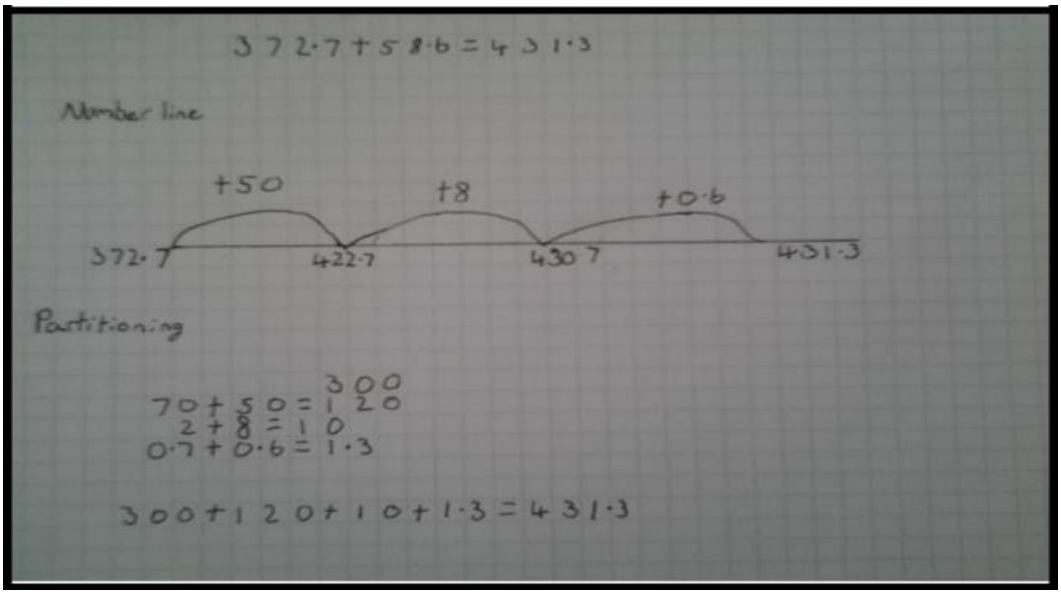
E.g. 357 + 268



Solve problems that include numbers with decimals.
E.g. 8.5 + 3.6



In Key Stage 2, pupils could continue to represent their calculations with a number line and use partitioning to solve addition problems.



Subtraction -

Definition

Subtraction is the process or skill of taking one number or amount away from another or **finding the difference** between two numbers.

Early learning

Using a range of practical resources and real-life contexts, pupils develop their understanding of the concept of subtraction through counting activities. They then use pictures/diagrams to represent the calculation.

E.g. There are 3 birds. 1 flies away. How many are **left**?

3-1=2

Mental Calculations

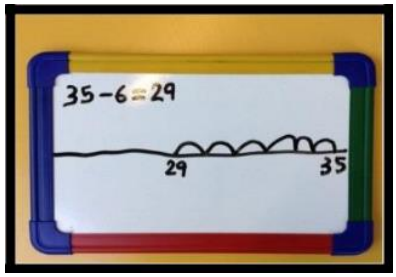
- Counting forwards and backwards in ones, twos, fives, tens etc.
- Reordering
- Partitioning: counting on or back.
- Partitioning: bridging through multiples of 10.
- Partitioning: compensating.
- Partitioning: using near doubles.
- Partitioning: bridging through 60 to calculate a time interval.
- Using addition as the inverse of subtraction.

Written Methods

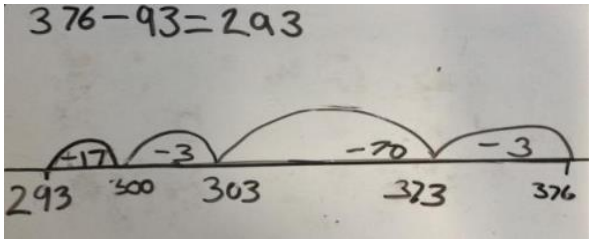
Number track/lines to count back or take away.

Use a number line to take away beginning with a number track e.g. 9-3.

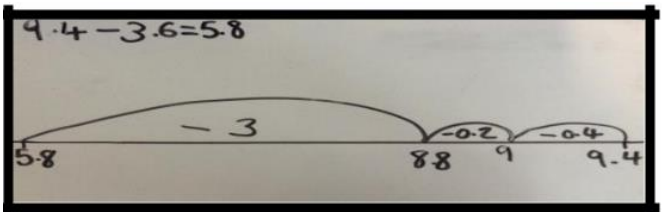
Progress to an empty number line when subtracting one digit from two-digit numbers while counting backwards e.g. 35–6.



Then progress to subtraction of two-digit numbers from three digit numbers e.g. 376- 93.

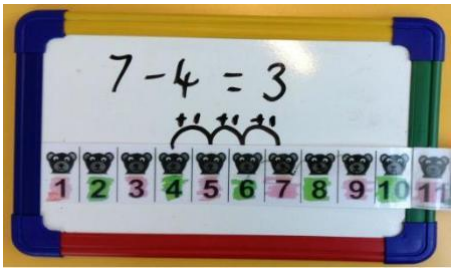


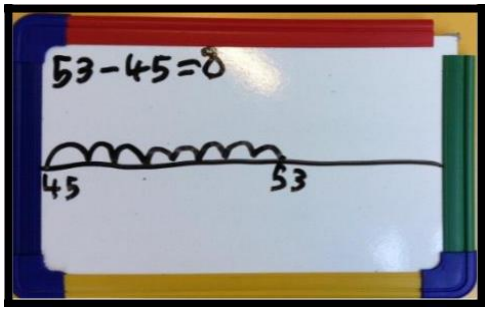
Solve subtraction problems that involve decimal numbers e.g. 9.4 -3.6.



<u>Number track/lines to find the difference or count up:</u>

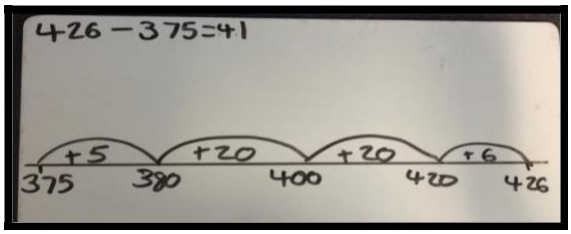
Use a number track to count up to find the difference e.g. 7-4.

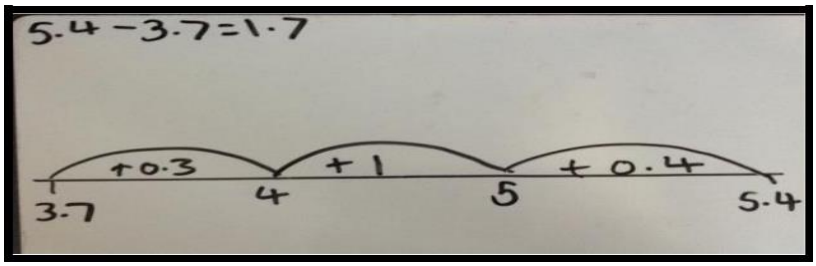




Progress to an empty number line to find the difference between two-digit numbers e.g. 53 – 45.

Then progress to subtraction of two-digit numbers from three digit numbers e.g. 426-375.



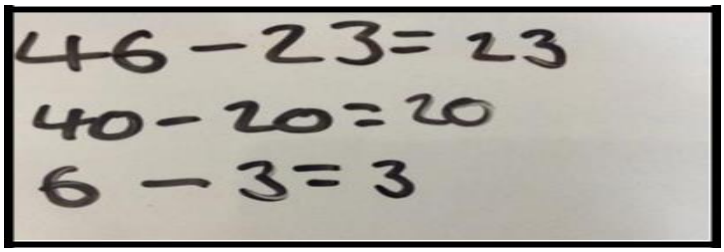


Solve problems involving decimal numbers e.g. 5.4 - 3.7.

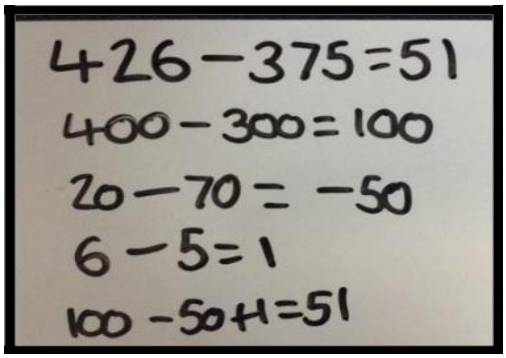
Partitioning

Partition the number e.g., into tens and ones.

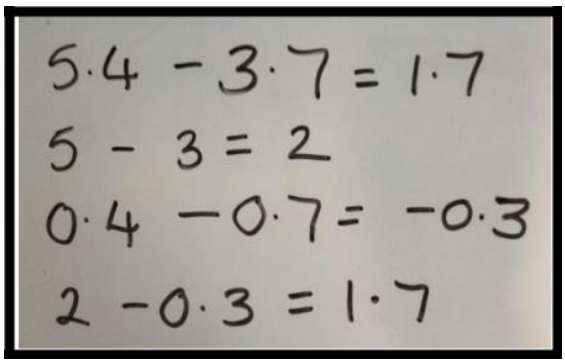
46-23=23



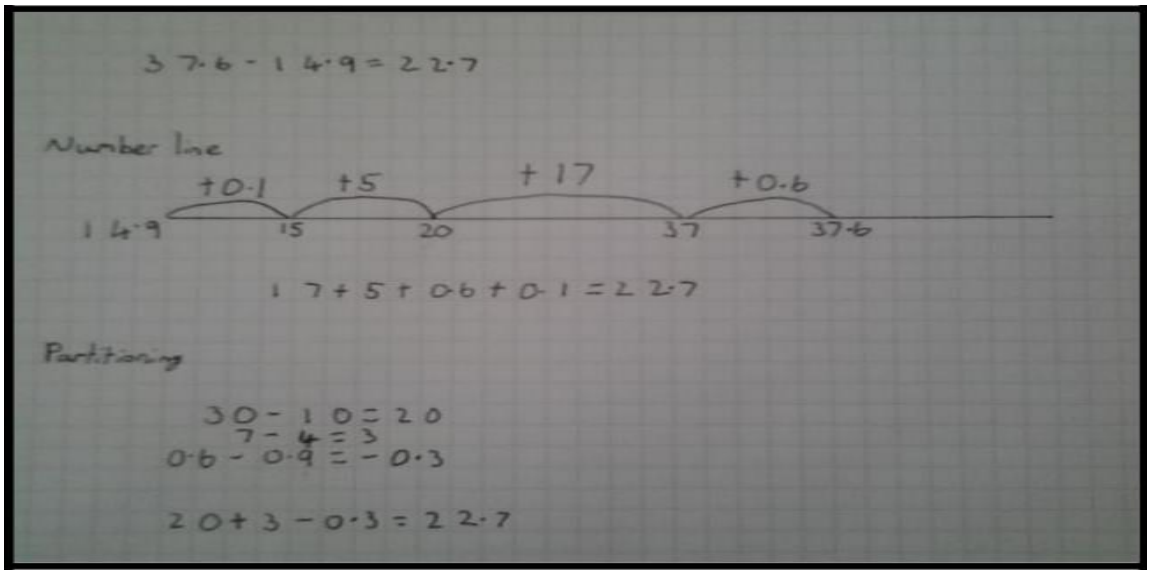
Partitioning involving negative numbers e.g. 426 – 375.



Solve problems involving decimal numbers e.g. 5.4 – 3.7.



In Key Stage 2, pupils will show progression in their mathematical thinking.



Multiplication X

Definition

Multiplication is the **product** of two numbers or repeatedly adding the same set of number as many times as the other number. Therefore, 3 multiplied by 4 is 4 lots of 3, or 3 added repeatedly 4 times. It is an inverse operation of division.

Early learning

Pupils are given an opportunity to manipulate and experience a range of resources in real-life contexts and through role play. They are encouraged to solve real-life problems e.g. If one pair of welly boots = 2 then 3 pairs = 6

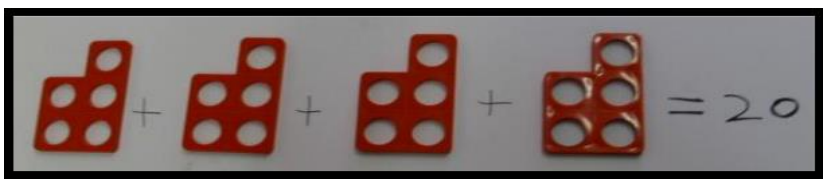
They are encouraged to draw pictures and represent their mathematical thinking through various representations e.g. bead strings, Numicon and cubes.

Finding doubles:

Mental calculations

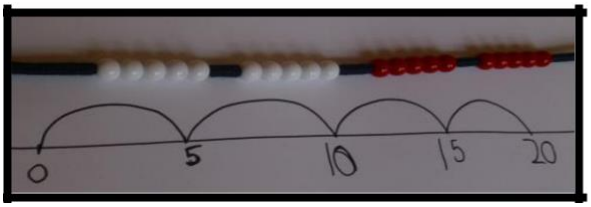
- Counting forwards and backwards in equal steps e.g. in 2's, 5's, 10's
- Repeated addition
- Rapid recall of multiplication facts
- Partitioning
- Secure understanding of place value.
- Multiplying and dividing by 10, 100 and 1000
- Doubling and halving
- Using division as the inverse of multiplication.

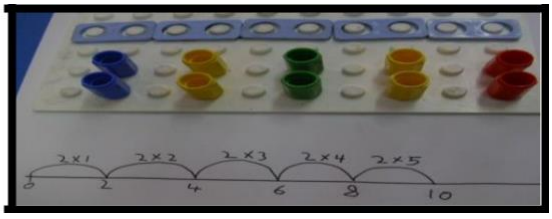
As pupils begin to be able to recall certain multiplication facts, they should be encouraged to develop strategies that allow them to work out other facts from the ones they know. Pupils will develop fluency with reasoning.



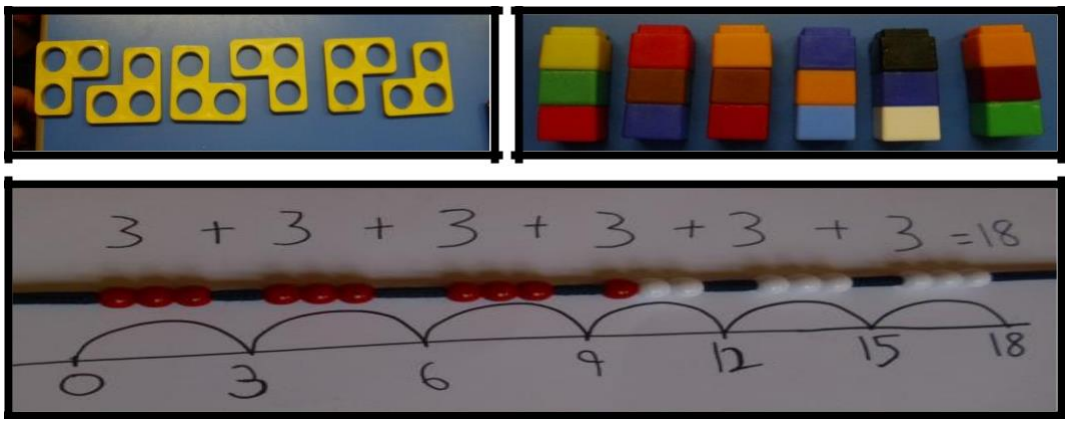
Written methods

Number lines



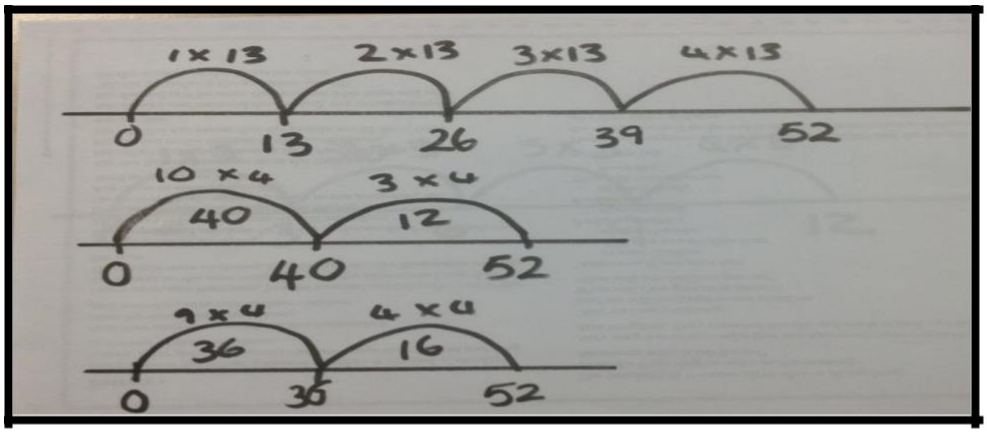


Use concrete representations such as bead strings, Numicon or cubes etc. to make sets or groups of various sizes. Use number lines alongside other mathematical equipment to represent repeated addition counting in regular steps of various sizes e.g. to calculate 3 x 6.



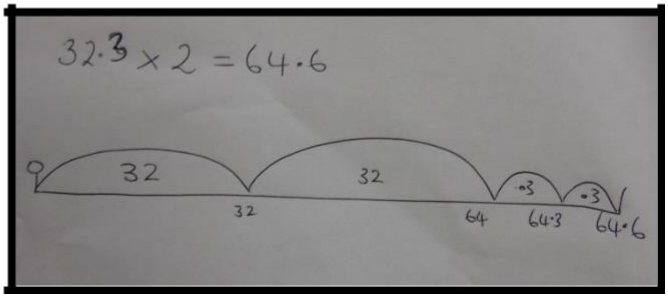
Partitioning

Encourage children to experiment with partitioning e.g. 13 x 4.



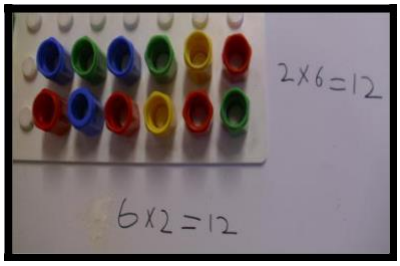
Multiply numbers with decimals e.g.

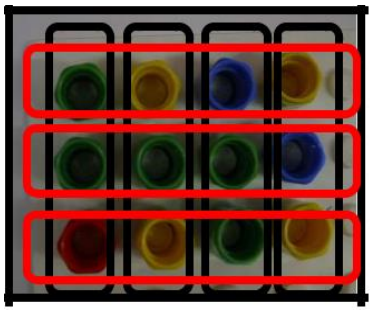
32.3 x 2 = 64.6.



Arrays

Arrays are an essential representation to solve multiplication problems. Pupils can count groups of objects in each row or column to find the product. They will be given a number of objects to arrange in an array of various dimensions and also be given an unknown number of objects to count by arranging in arrays.





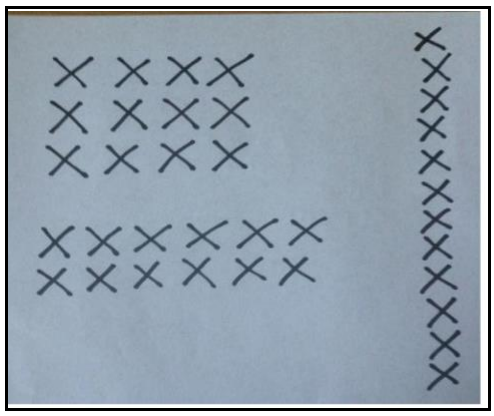
E.g. 3 lots of 4 = 4 groups of 3

4 x 3 = 3 x 4

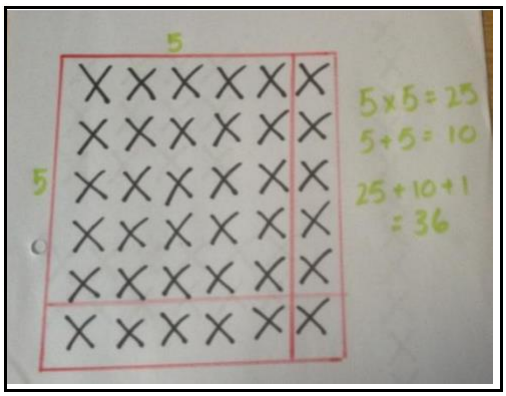
4+ 4 + 4 = 3+ 3 +3 +3=12

Use of concrete representations should lead to drawing pictures and arrays.

e.g. Arrange 12 counters into an array



e.g. Find the unknown product by making groups in the array



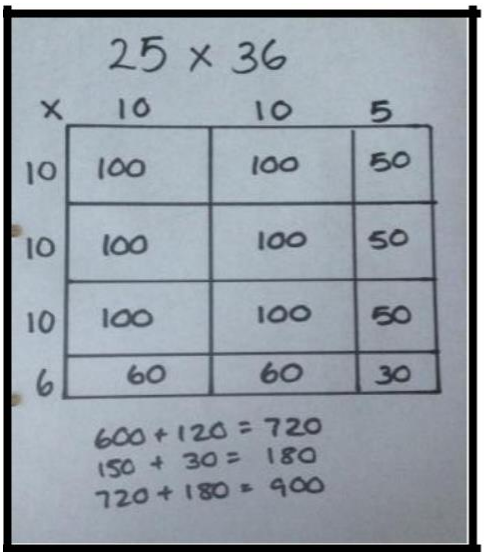
Grid method

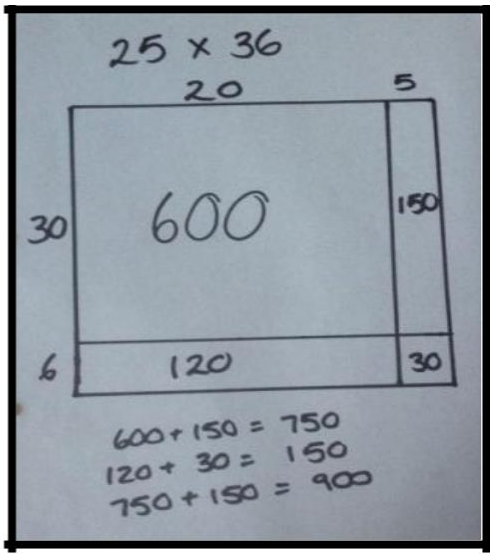
Experienced learners will adopt the grid method. Grids should be proportional and children should be allowed to choose how to partition each number, not just into tens and ones. Children will start by multiplying a two-digit by a one-digit number.

e. g 35 x 7 = 245

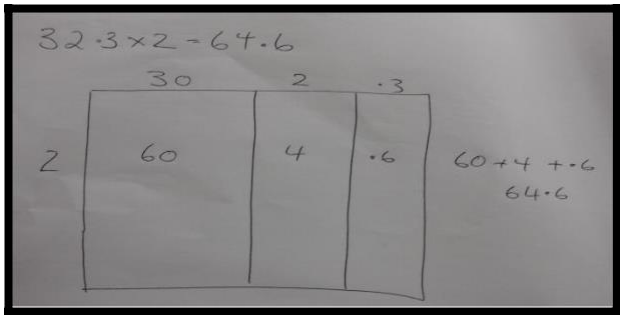
x	30	5
7	210	35

Multiply two-digit numbers e.g. 25 x 36.





Multiply numbers with decimals e.g. 32.3 x 2 = 64.6

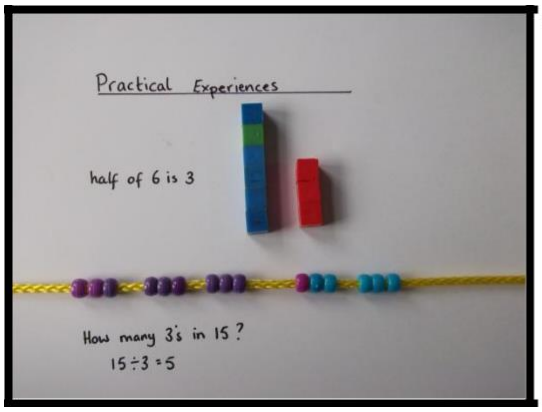


Division ÷

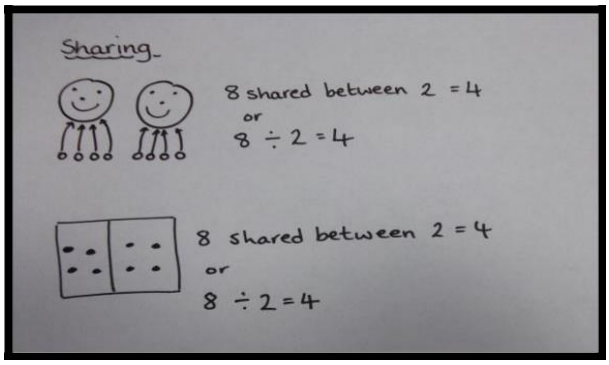
Definition

Dividing is a quick way of subtracting several lots of the same number or quantity or splitting it up into equal groups. Multiplying and dividing are the inverse or opposite of each other.

Early learning



Pupils should have many practical experiences of sharing objects e.g. sharing between 2 people or finding ½ of a group of objects. Pictures should be introduced as a next step to represent this.



Drawings and diagrams should be increasingly used to represent and demonstrate **sharing**.

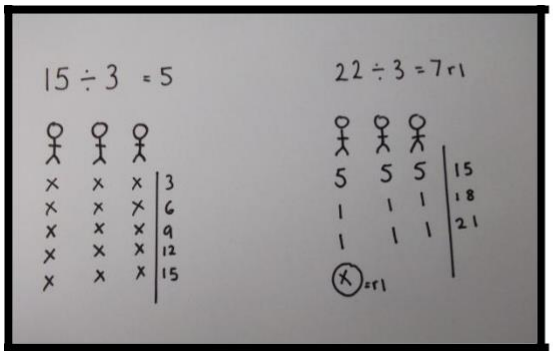
Mental calculations

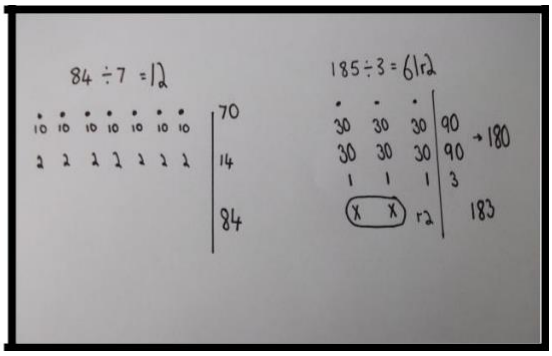
- Counting forwards and backwards in equal steps e.g. 2's, 5's, 10's.
- Rapid recall of multiplication facts.
- Partitioning
- Secure understanding of place value.
- Multiplying and dividing by 10, 100 and 1000.
- Doubling and halving.
- Using multiplication as the inverse of division.

Written methods

Sharing

Progressing from the drawings and diagrams listed above, pupils can share by partitioning or 'chunking.' Here, sharing can be recorded as jottings and numbers.

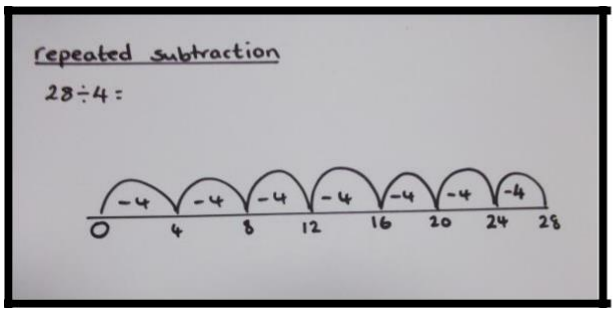


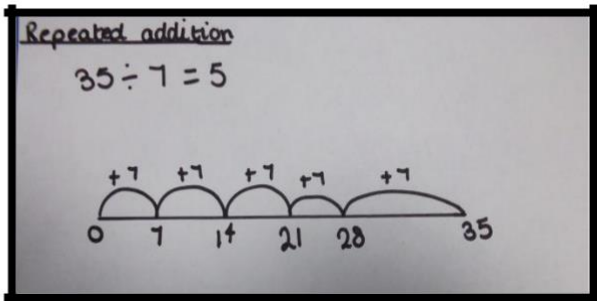


Grouping

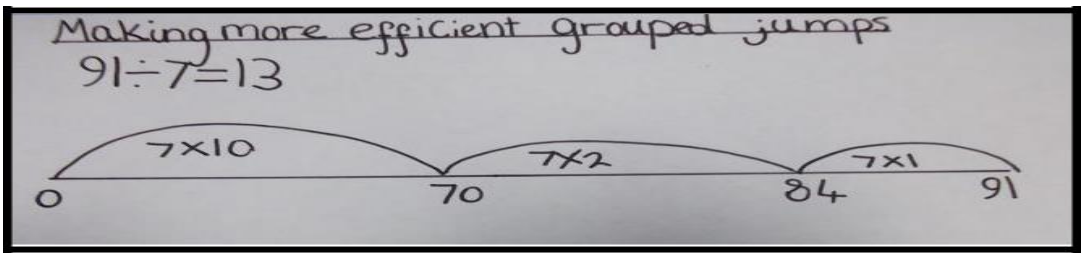
Using a number line:

Number lines should be introduced to help record grouping strategies. This would involve the principle of repeated subtraction (and repeated addition as an alternative strategy).

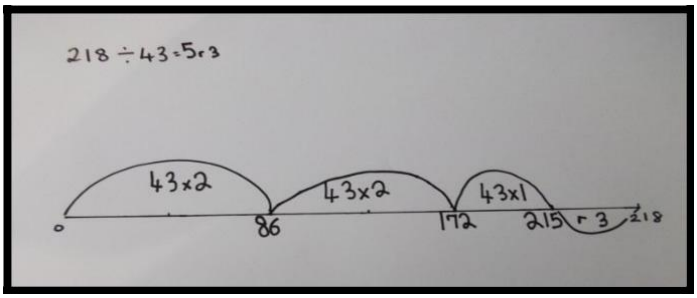




Pupils will progress to more efficient methods showing greater conceptual understanding.



Where division leads to a **remainder**, the remainder can be shown under the number line as illustrated here.



Algebra

What is Algebra? Why is it important?

Algebra is a way of thinking and a set of concepts and skills that enables pupils to generalise, model, and analyse mathematical situations. Algebra provides a systematic way to investigate relationships, helping to describe, organise, and understand the world. Although learning to use algebra makes students powerful problem solvers, these important concepts and skills take time to develop. Its development begins early and should be a focus of mathematic instruction from EYFS through all key stages.

The use of the equals sign =

In maths, equality (=) means balance between two sets and inequality (≠) means an imbalance.

Algebra requires pupils to solve simple equations that involve addition, subtraction, multiplication and division with a deeper understanding of the 'equals' symbol. Using concrete resources to start with, they should be able to explore the equality and inequality of values of numbers.

4	+	2	=	?	+	3
a	**+**	**b**	**=**	?	**+**	**c**

A helpful pedagogy to use is, **"What's the same and what's different?"** on both sides of the equation.' There must be an opportunity to experience some examples of inequality to appreciate equality in a greater sense.

7 ≠ 8 - 3 (not equal ≠)

In early number work, children should be encouraged to **look for patterns** and **generalise** by drawing out similarities.

10 = 1 + 9

11 = 2 + 9

12 = 3 + 9

Add one to both sides of the equation to balance them.

b = a + c

b+1 = (a+1) + c

Later, they should be encouraged to complete the sequence to the **nth term**.

e.g. 5, 8, 11, 14, 17,…… so the nth term = 3n + 2

Pupils should be given the opportunity to find the unknown or the missing number in all areas of calculations.

E.g. If each banana costs 2p more than an apple, what is the cost of 3 bananas?

Price of one apple = b

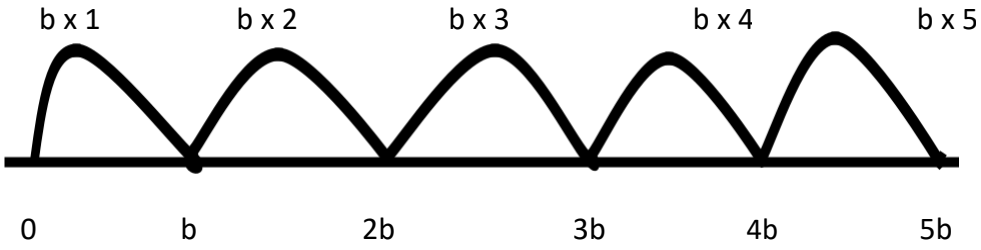
A banana would cost b + 2

3 bananas would cost 3(b + 2) = 3b + 6

	b	2
3	3b	6

What is the cost of 5 apples?

5 apples would cost 5 x b or 5b



Pupils should be encouraged to **make connections** e.g.

3 x 2 + 3 x 4 = 3 x 6

Find the missing numbers to solve problems e.g.

10 ÷ 5 = 20 ÷ a

10/5 = 20/a

Multiply both sides of the equation by 5: 5 x 10/5 = 20/a x 5

Then multiply both sides of the equation by **a**: 10a = 100/a x a

Finally, divide both sides of the equation by 10: a = 10

Pedagogical Approach

- Developing pupils understanding of number and place value is essential and should be explored daily.

- The strategies chosen should aim to develop pupils conceptual understanding of calculation, with a small steps approach in lessons, thus ensuring all children have a secure grasp of the skill being learned before moving on. Rapid graspers should be challenged with in depth reasoning & problem-solving, rather than simply moving on to "bigger" numbers.

- Models, images and resources (representations) in line with the CPA approach should be used throughout all key stages.

- Pupils should be encouraged to develop independence, and to select and use resources to support their learning.

- Practical activities should be a regular feature of maths lessons.

- Opportunities to work on the same problem/activity within mixed ability groups should be encouraged, wherever and whenever possible, giving all children an equal chance to develop their mathematical skills. Appropriate scaffolding may be required for some children to achieve this.

- It is more effective to provide pupils with one question to practice the same skill in depth, rather than lots of different questions.

- Problem solving and reasoning should be integral to every maths lesson.

- Pupils should be encouraged to take risks, make mistakes, and learn from their experiences.

- Teachers will explore misconceptions with pupils in order to deepen their understanding.

Fluency, Reasoning and Problem-Solving

What does fluency, reasoning and problem-solving look like in solving calculation questions?

These are the three aims from the 2014 Mathematics National Curriculum which are to ensure all pupils:

- become **fluent** in the fundamentals of mathematics, through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.

- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The 2014 Mathematics National Curriculum states that *'Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas…and that all pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems.'*

Examples of fluency, reasoning and problem solving:

8 x 5 = 40

Starting with this problem, pupils who demonstrate good fluency, reasoning and problem-solving skills are able to use this fact to create others such as:

5 x 8 = 40	8 x 5 = 20 x 2	5 x 8 = (5 x 10) – (5 x 2)
40÷5=8	(2 x 4) x 5 = 10 x 4	0.8 x 0.5 = 0.4
40÷8=5	16 x 2.5 = 40	5 x 8 = 10 x 4
8 x 50 = 400 80 x 50 = 4000	40 x 8 ≠ 5 5 x 8 =8+8+8+8+8	2^3x √25 = 40 = 8 x 5 40 = 8 x 5

Use of ICT

ICT plays an important role in the support and development of learning in mathematics. Technology can be used to promote the thinking and reasoning skills needed for problem solving within mathematics as well as increasing pupils understanding of arithmetic operations and numerical relationships.

The use of technology should be carefully planned to advance learning within the classroom and it should not be used as a replacement for basic understanding. Technology should not replace the need for pupils to develop efficient and accurate methods for both mental and pencil and paper calculations as well as performing sensible estimations.

ICT resources that may be used to support the teaching and learning of calculations could include calculators, computers, tablets and interactive whiteboards.

Useful Websites

National Centre for Excellence in the Teaching of Mathematics www.ncetm.org.uk

NRICH - enriching mathematics http://nrich.maths.org

White Rose https://whiterosemaths.com/

National Numeracy http://www.nationalnumeracy.org.uk

National Strategy Archive – Interactive Teaching Programs

http://webarchive.nationalarchives.gov.uk/20110809101133/http:/www.nsonline.org.uk/search/primary/results/nav:49909

BBC Bitesize http://www.bbc.co.uk/bitesize

Woodlands Junior School http://www.woodlands-junior.kent.sch.uk

Twinkl https://www.twinkl.co.uk/

Primary Resources http://www.primaryresources.co.uk/

Classroom Secrets https://classroomsecrets.co.uk/

National Stem Centre http://www.nationalstemcentre.org.uk/elibrary

Maths is Fun http://www.mathsisfun.com

Times Educational Supplement http://www.tes.co.uk/teaching-resources

Mathematics in Education and Industry (MEI) http://www.mei.org.uk/

Maths Zone http://mathszone.co.uk/

Glossary

Add +	Add, total, sum, more, plus, increase, altogether
Algebra	a symbol representing a number
Arrays	A rectangular representation where each row and column must have the same number of objects or pictures.
Commutative	In addition, numbers can be added up in any order and the total remains the same e.g. a + b = b + a.
Divide ÷	Dividing is a quick way of subtracting several lots of the same number of quantity, or splitting it up into equal groups.
Divisor	The divisor is the number you divide by e.g. in 6÷3 the divisor is 3.
Equals	the same value as, equivalent, balance
Equation	A number sentence that uses letters or symbols to replace digits, a statement where two mathematical expressions have the same value.
Generalise	Look for a general pattern that will help to solve a related problem.
Grid	A way to organise a multiplication or division calculation where the number being divided is partitioned. The partial products are shown in the grid. See diagram in multiplication and division sections.
Grouping	Where a set of objects or a number is grouped into an already established number until no more groups can be made. E.g. 35 sweets shared between 7 friends would make 7 groups of 5.
Inverse	Opposite or reverse operations, e.g. 16 – 7 = 9 so 9+7=16
Multiply x	So many groups of, lots of, and sets of. Times, find the product of.
Pedagogy	Teaching method used e.g. asking questions, encouraging to look for patterns etc.
Quotient	A quotient is the whole number of times you can divide one number by a number.
Remainder	If you can't divide a number exactly you have an amount left over – this is called the remainder--r.

Repeated addition	Repeated addition is the process of grouping. Where a number is repeatedly added from 0 to the target number e.g. repeatedly adding 5. For larger numbers, multiples of e.g. 5 can be repeatedly added.
Repeated subtraction	Repeated subtraction is the process of grouping. Where a number is repeatedly subtracted from the total e.g. repeatedly subtracting 5 from 35 (7 times). For larger numbers, multiples of e.g. 5 can be repeatedly subtracted.
Sharing	Where a set of objects or a number is shared equally into a given number of sets. E.g. 28 sweets are shared into 7 equal piles.
Subtract -	Minus, take away, find the difference, count how many left, find that many fewer than before.

Monitoring and Impact

This policy is a working document that will be updated regularly to evaluate its impact.

- ✓ All staff will adopt this policy and participate in the continuous development and progression of this document.
- ✓ All data will be monitored and moderated in a consistent manner.
- ✓ The school will hold regular meetings with parents and events and, as part of these, the maths and calculation policies will be explained.
- ✓ Staff and parent questionnaires will be sent out and analysed periodically to monitor the success of dissemination.
- ✓ CPD events for all staff will be facilitated either through the CMAT or independently.
- ✓ The transition process will be monitored by members of the Senior Leadership Team with the Headteacher in school.
- ✓ This policy will be available to view on the school website, under the Maths Curriculum tab.

What if?

Frequently asked questions considered by the Maths Subject Leader:

What if pupils prefer a different method?

Allow the child to use their method if they show conceptual understanding and can consistently use the method accurately. However, the teacher will follow the school policy. One of the aims of the 2014 Mathematics Curriculum encourages pupils to be fluent in their approach to calculation strategies. Therefore, a variety of methods are necessary.

What if pupils join the school with a different method?

See above

What if parents don't like the methods adopted?

Explain the reasoning behind the chosen strategies and encourage parents to become involved in their child's mathematical development. Provide parents with a copy of the Calculations Policy, making reference to the research used.

What if parents teach another method?

See question 1.

What if all the teaching staff are not able to support the methods?

Provide appropriate training and support.

What if Ofsted ask why the school is not moving the children onto the next stages sooner?

The National Curriculum for Mathematics 2014 states that children should only progress to the next stage once they have secure understanding. Therefore, based on our research and experience in school, these are the most efficient strategies for our children to be successful and fluent in calculations.

It goes onto say: 'Within each key stage, schools therefore have the flexibility to introduce content later than set out in the programme of study.'

Continual Professional Development (C.P.D)

It is important for all staff to have a clear understanding of each of the calculation methods outlined in this policy. CPD will play a central role in introducing new ideas and clarifying any misunderstandings. This will lead to consistent practice within the school.

This calculations policy will be reviewed and monitored by the maths leader and Headteacher annually in order to refresh ideas and maintain its effectiveness.

It is important to make sure that all staff, including TAs, supply teachers and Cover Supervisors, are aware of which methods should be taught and how they should be taught. All teaching staff should follow this policy when teaching mathematics in all areas of the curriculum. Resources should be displayed in classrooms to support this.

Sessions may include:

- Reviewing each calculation strategy
- Discussing which practical resources support each method.
- Sharing ideas on how the calculations have been included in contextualised opportunities.
- Discussing ideas on how to assess the use, understanding and enjoyment of calculations within school.

The Maths Subject Leader will work with teachers and parents to ensure that they have been provided with suitable materials to support their children and have opportunities to speak to staff about calculation methods or watching staff model these strategies.

Research

'A feature of strong practice in (the maintained) schools is their clear, coherent calculation policies and guidance, which are tailored to the school's context. They ensure consistent approaches and use of visual images and models that secure progression in pupils skills and knowledge lesson by lesson and year by year.' Good practice in primary mathematics – evidence from 20 successful schools (Ofsted 2011)

The following books and articles contain research that has been helpful to produce this calculation policy:

Anghileri, J. (2000) Teaching Number Sense. London: Continuum

Boaler, J (2009) The Elephant in the Classroom: Helping Children Learn and Love Maths. London: Souvenir Press Ltd.

Borthwick, A. and Harcourt-Heath, M. (2007) Calculation Strategies used by Year 5 Children. Proceedings of the British Society for Research into Learning Mathematics, 27(1), 12-17.

Borthwick, A. and Harcourt-Heath, M. (2014) Calculation Strategies used by Year 8 Children. Proceedings of the British Society for Research into Learning Mathematics.

Carpenter, T, Loef Franke, M, Levi, L (2003) Thinking Mathematically. Portsmouth: Heinneman.

Cockburn, Anne (2008). Mathematical Misconceptions. London: Sage Publications Ltd.

Cotton, T (2010) Understanding and Teaching Primary Mathematics. Harlow: Pearson Education Ltd

Haylock, D (2010) Mathematics Explained for Primary Teachers (4th edition). Harlow: Sage Publications Ltd

Rowland, T, Turner, F, Thwaites, A, Huckstep, P (2009) Developing Primary Mathematics Teaching. London: Sage Publications Ltd

Skemp, R. (2012) Mathematics in the Primary School [online]. London: Routledge. http://reader.eblib.com.

Thompson, I (1999) Issues in Teaching Numeracy in Primary Schools. Maidenhead: Open University Press.